

On the Interaction of a Weak-R Type Ionization Front and a Contact Discontinuity

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A numerical solution is given to the problem of a weak R type ionization front approaching a contact discontinuity across which the density increases. During the interaction it is found that there are three possible configurations produced; the weak-R type ionization front becomes (a) weak-R type but slower for $\rho_1/\rho_0 < 1.15$, (b) strong-D type for $1.15 < \rho_1/\rho_0 < 3.4$ or (c) weak-D type for $\rho_1/\rho_0 > 3.4$, where the original ionization front travels through a region of initial density ρ_0 and across the contact discontinuity the density becomes ρ_1 . A fine spacial mesh length is chosen so that the structures of the resulting ionization fronts and shock waves may be identified clearly at any time.

1. Introduction

The interaction between a weak-R type ionization front and a contact discontinuity is studied numerically.

A plane contact discontinuity separating two regions of neutral hydrogen each at constant (though different) densities is subject to ionizing radiation from distant stars. The radiation causes the neutral hydrogen to be ionized and the narrow transition between ionized and non-ionized states is called an ionization front. The time evolution of ionization fronts through HII regions of continuous density has been studied by many authors, both analytically and numerically (see, for instance, Goldsworthy [1], Matthews [2], Tenorio-Tagle [3]). During the early stages of the flow, before the ionization front reaches the contact discontinuity, the ionization front slows down from very fast weak-R type to slow weak-R type, then to strong-D type, and finally to weak-D type, in the latter stages a shock wave travels ahead of the ionization front. The positioning of the contact discontinuity affects which type of ionization front interacts with the density jump.

In this paper a weak-R type ionization front of speed approximately $3.5c$ (where c is the sound speed in the ionized gas) interacts with the contact discontinuity across which the density increases in the direction of motion of the ionization front. The problem is simplified by assuming that the radiation impinges normally on the discontinuity, resulting in a plane ionization front and thus retaining plane symmetry in the equations of motion. The full

non-steady equations of motion governing the flow are solved numerically.

Previous studies on problems involving interactions have been done by Marsh [4] and Tenorio-Tagle [5]. Marsh investigated analytically the interaction between a weak-R type ionization front and a contact discontinuity. The early and later stages of the evolution are unsteady; however, Marsh found that under certain circumstances an intermediate time scale exists during which the flow may be taken as quasi-steady and solved for problems in which this approximation is valid. Marsh found that a relationship involving the HII region radius and the ionization front thickness must be satisfied for validity of the method. Such a restriction is not necessary in this paper where the full unsteady flow solutions are given for different density ratios.

Tenorio-Tagle included an interaction between an expanding shock wave and a contracting weak-D type ionization front in spherical symmetry, for the time evolution of a globule immersed in an HII region. Interactions of this type, i.e. between shock waves and ionization fronts of different types, will be the subject of a future paper.

2. The Mathematical Model and the Equations of Motion

The mathematical model consists of a plane ionization front, produced by plane parallel ionizing radiation from distant stars, travelling through a region of neutral hydrogen of constant density ρ_0 . This ionization front then interacts with a contact discontinuity across which the density jumps to a constant value ρ_1 , as shown in Figure 1.

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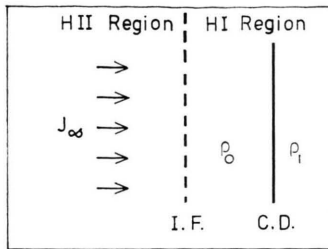


Fig. 1. The interaction of an ionization front with a contact discontinuity. The incoming ionizing radiation is shown by the arrows. The dotted line represents the edge of the HII region.

The equations describing the planar symmetric, unsteady motion of a gas subject to ionizing radiation have been discussed fully elsewhere (see, for instance, Goldsworthy [1]). These equations are solved numerically in the region of constant density $\rho_0 = 10 \text{ atom/cm}^3$ with downstream boundary condition $u=0$ at the "stars". For a full discussion of the numerical technique employed see Tenorio-Tagle [3].

3. The Interaction of a Weak-R Type Front

a) The Initial Conditions

The flux of ionizing radiation per unit area, per unit time (J) arriving at the ionization front was chosen so that a weak-R type ionization front of speed $3.5c$ approaches the contact discontinuity. With a value of $T_* = 60,000^\circ\text{K}$, an HII region temperature of 9035°K is obtained giving a value of the sound speed $c = 12.21 \text{ km/sec}$.

b) Ionization Front and Shock Wave Identification

The spacial mesh length (Δs) must be chosen carefully so that the ionization front structure and shock wave structure are easily identified. Hill and Marsh [8] have shown that, to ensure correct solutions in the D' type phase of the motion, several meshes per recombination length are necessary. With the isothermal sound speed c as a typical velocity the recombination length $l_c = ct_R$ where t_R is the relaxation time $MTc^{3/4}/\rho_0\beta_0$.

The region in which the fractional ionization x changes from 0.005 to 0.98 is called the ionization front and its position is taken to be the point where $x=0.5$. The velocity of the ionization front is calculated by taking the distance travelled in one

time step and dividing by Δt . Having found the speed of the ionization front and the gas velocities on either side of the front, which are obtained from the numerical solution, the ionization front is classified in the usual way, i.e. if the velocity of the front relative to the neutral gas is supersonic it is termed "R type", and "D type" if it is subsonic; the terms "weak R" and "strong D" or "weak D" are used if the velocity of the front relative to the ionized gas is supersonic or subsonic respectively.

Shock waves are expected to exist after the interaction and are recognized by a discontinuity which moves supersonically relative to the gas in front and subsonically relative to the gas behind. The position of the shock wave is defined to be the point where the density has its greatest value, from which we can estimate its speed.

c) The Numerical Results

Interactions are considered for various values of the ratio of the initial densities $\rho_1/\rho_0 (>1)$. The solution can be considered in two phases: (I) the interaction: a time scale during which the discontinuous jump in density smooths out producing a new configuration of ionization front and shock fronts and (II) the expansion and evolution of this new set of fronts.

(i) Description of the Solution for $\rho_1/\rho_0 = 2$

I. The Interaction. Figure 2 shows the variation in u (velocity), x (degree of ionization) and ρ/M (number density) at four times $t_1=0$, $t_2=1.4 \times 10^3$ years, $t_3=2.8 \times 10^3$ years and $t_4=4.2 \times 10^3$ years. $t=0$ is chosen just before the ionization front reaches the contact discontinuity which is positioned at $r=0$ for convenience.

The density and velocity profiles indicate quite clearly the transition of the ionization front from weak-R type to strong-D type which is preceded by a strong shock. The interaction is considered complete when this shock wave is fully developed. Figures 2(a) and 2(d) represent the initial and final configurations respectively. An isothermal shock moving away from the ionization front and into the HII region is quite clearly shown in Figure 2(d) with a rarefaction wave just behind the ionization front. Figures 2(b) and 2(c) show the development of this rarefaction wave and isothermal shock. This

shock wave in the HII region is represented on the diagram by the dotted lines on the velocity curves. The curve for x shows the position of the ionization front at each time.

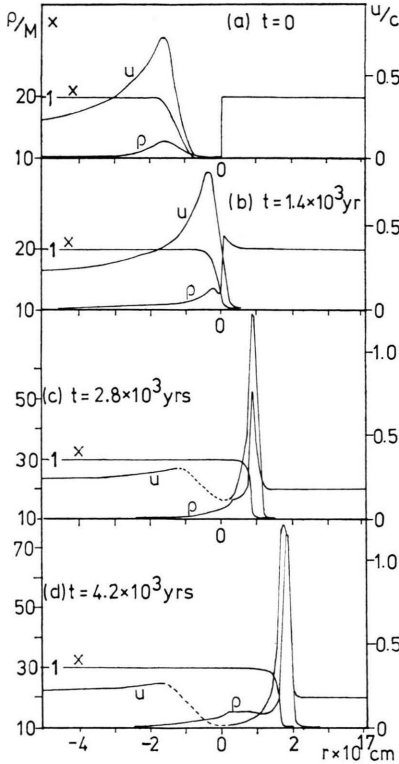


Fig. 2. The number density, velocity and degree of ionization profiles before 2(a), during 2(b) and 2(c) and after 2(d) the interaction for $\rho_1/\rho_0 = 2.0$. The dotted portion of the velocity curve shows the development of the isothermal shock in the HII region.

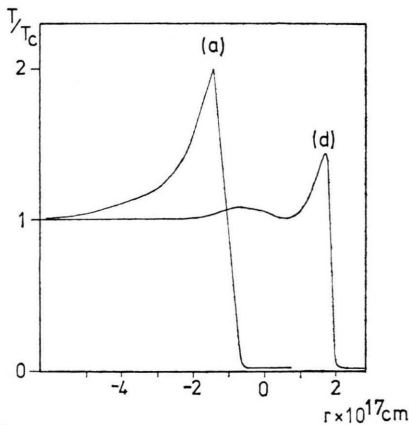


Fig. 3. The temperature profile before (a) and after (d) the interaction.

The time scale for the interaction is 4.2×10^3 years and the speed of the strong-D type ionization front after the interaction is approx. $1.42 c$.

The isothermal shock front in the HII region has speed $0.86 c$.

Figure 3 shows the temperature distribution T/T_c at times $t=0$ and after the interaction ($t=4.2 \times 10^3$ years). The bump in the curve corresponds to the position of the contracting isothermal shock front in the HII region.

II. Continuing Evolution. The ionization front, which is now strong-D type, slows down and eventually becomes weak-D. Figures 4(a) and 4(b) show the velocity, number density and degree of ionization profiles at times $t=7.4 \times 10^3$ years (strong-D) and $t=1.5 \times 10^4$ years (weak-D). The

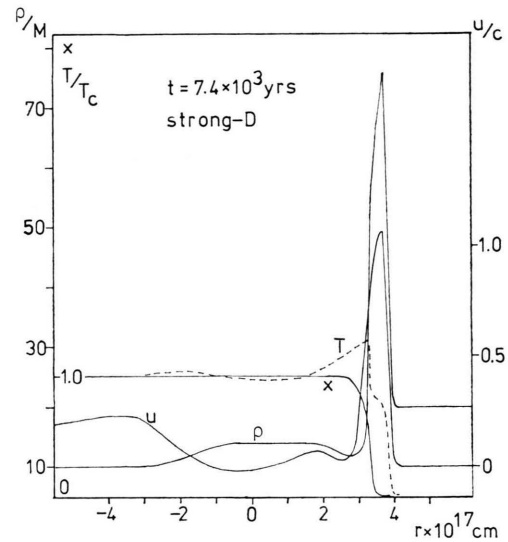


Fig. 4(a). Strong-D phase of the evolution.

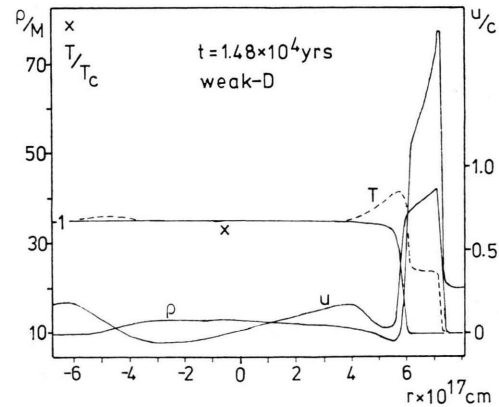


Fig. 4(b). Weak-D phase of the evolution.

isothermal shock front in the HII region gradually increases in speed and strength as the region evolves. Figure 5 shows the variation of shock speed with time.

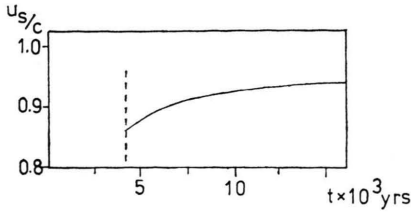


Fig. 5. Velocity of the isothermal shock front in the HII region as a function of time. The interaction of the ionization front and contact discontinuity occurs between $t=0$ and the dotted line.

(ii) Solutions for other Values of ρ_1/ρ_0 .

Similar computations to those described above were carried out for different values of ρ_1/ρ_0 . The transition of the ionization front from weak-R type to (a) weak-R type but faster, (b) weak-R type but slower (c) strong-D type or (d) weak-D type depends on the size of the ratio ρ_1/ρ_0 . Case (a) is not considered here but will occur for $\rho_1/\rho_0 < 1$. Figure 6

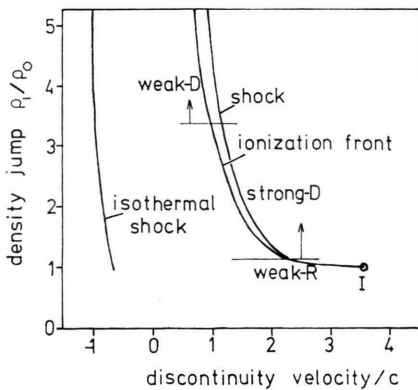


Fig. 6. The velocity of the discontinuities as a function of the density jump ρ_1/ρ_0 .

illustrates the fate of the weak-R type ionization front for different density jumps and gives the resulting speeds of the discontinuities after the interaction. The point I represents $\rho_1/\rho_0 = 1$ and no contact discontinuity is present. At about $\rho_1/\rho_0 = 1.15$ the weak-R type ionization front becomes strong-D type and a strong shock wave travels ahead into the undisturbed gas. For $\rho_1/\rho_0 > 3.4$ the weak-R type front becomes weak-D.

As ρ_1/ρ_0 increases the speed and strength of the isothermal shock in the HII region increase, whereas the rarefaction wave following the ionization front decreases in strength. Cases (b) and (d) are illustrated in Figures 7 and 8.

Figure 7 shows the variation in u , ρ , x and T for $\rho_1/\rho_0 = 1.1$. The time scale for the transition is

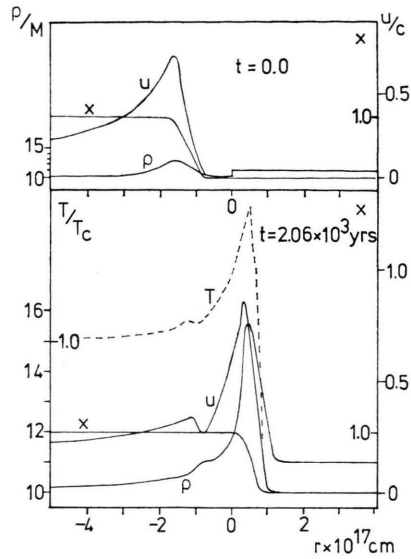


Fig. 7. Before and after the interaction for $\rho_1/\rho_0 = 1.1$. The resulting ionization front is weak R type.

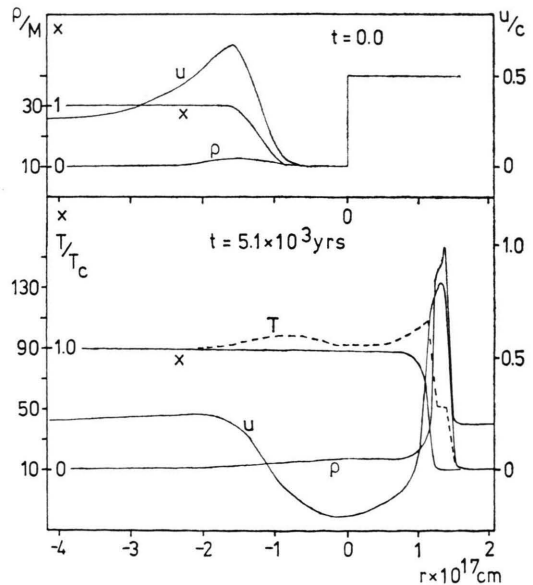


Fig. 8. Before and after the interaction for $\rho_1/\rho_0 = 4.0$. The resulting ionization front is weak-D type.

2.06×10^3 years and the resulting weak-R type ionization front has speed 2.42 c.

Figure 8 shows the variation in u , ρ , x and T for $\rho_1/\rho_0 = 4.0$. The time scale for the transition is 5.1×10^3 years and the resulting weak-D type ionization front has speed 0.84 c. This front is preceded by a strong shock front with speed 1.02 c and the contracting isothermal shock in the HII region has speed 0.97 c.

Conclusions

The computations described in this paper have illustrated the various features of the flow as the interaction of a weak-R type ionization front and a contact discontinuity evolves. Just before the interaction, the ionization front is weak R type with speed 3.5 c. The flow in the HII region is small and the density increases slowly from the “stars”, at which $u = 0$, to the ionization front.

The interaction of the ionization front and the contact discontinuity produces three regions of flow; an outer undisturbed region of density ρ_1 , an inner fully ionized region which does not change appreciably during the time scale of the interaction and the interaction zone in which the initial density discontinuity is smoothed out. The boundaries of this zone are an isothermal shock front travelling towards the “stars” and an ionization front bound-

ing the neutral and ionized gas. When the ionization front is D' type it is preceded by a strong adiabatic shock. The following facts about the interaction zone have been deduced from the numerical data:

- a) The weak-R type ionization front becomes strong-D type for $\rho_1/\rho_0 > 1.15$ and weak-D type for $\rho_1/\rho_0 > 3.4$.
- b) The time scale for the interaction increases with ρ_1/ρ_0 as expected.
- c) The rarefaction wave just behind the ionization front decreases with strength as ρ_1/ρ_0 increases.
- d) The isothermal shock wave increases in strength and thickness as ρ_1/ρ_0 increases.

The continuing evolution of the model is also given. The ionization front continues to slow down, the isothermal shock, which travels into a region of decreasing density, increases in speed with a small increase in strength. Gas velocities in the HII region remain small e.g. in the weak-D phase for $\rho_1/\rho_0 = 2$, $u \approx 0.25$ c; this compares with $u \approx 0.5$ c for the normal evolution of a weak-R type front to a weak-D type front. One essential difference however is that the shock front which precedes the D type ionization front loses strength very quickly and is much closer to the ionization front.

Finally it is worth noting that the results obtained here are in good qualitative agreement with Marsh's analytical solution.

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